Chapter 4

Multiplication
**Multiplication** (When planning ensure you track forwards to year 1)

**Early Learning Goal 11**
Using quantities and objects, they add two single-digit numbers and count on or back to find the answer. They solve problems, including sharing, doubling and halving.

**Key vocab:** count in, double, halve, lots of, groups of, times, group in pairs, equal groups of.

**Key concepts**
Multiplication begins with counting patterns and contexts involving equal groups.
Objects can be added over and over again to make ‘more’.
Objects can be sorted into groups of the same number.
To get the total you count according to the number in the group.
All steps need to be taught through play as they are the precursors to multiplicative reasoning.
Children will be working in the concrete before moving towards the pictorial and abstract.

**Potential barriers/misconceptions**
Children inaccurate when displaying arrays of cubes/objects and so pattern is not clear.
Link not clear between the array and the seemingly abstract number given as the answer.
Children unable to place objects in equal groups.
Not secure with one to one correspondence counting in ones, therefore will be unable to count pairs accurately.
When counting orally in 10s: 60, 70, 80 follow a regular pattern which links to single digit numbers however 10, 20, 30 do not.
Conceptual understanding of ‘same’ and ‘different’ is not secure (both language and concept).

**Learning objectives**
See over page for exemplification.

**KS1 ready:**
Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

**Mental Maths** (can revisited throughout day once concept has explicitly shared)

- Count in tens (recite the sequence ten, twenty, thirty... one hundred.) Do the same backwards.
- Count on and back in tens from a given tens number.
- Say the tens number that goes before or after a given tens number. (When you count in tens, what number comes before 60? 90?)
- Count from a given tens number and stop at another. (count on in tens from 20 and stop at 70, count back in tens from 60 and stop at 30)
- Count around in a circle of children, starting with Abdul on 20, who do you think will say 70?
- Understand odd and even numbers linked to getting ‘into pairs’.
- Count pairs: children, socks, animals in the ark, eggs in an egg box

**Example Questions**
Give everyone two biscuits from the jar.
Can you count the spots on each side of your butterfly? Does it have an equal number of spots on each side?
I will clap where there is a number missing. 1, 2, 3, (clap) 5. Tell me the missing number 2, 4, 6, (clap), 10 Tell me the missing number 10, 20, 30, (clap), 40. Tell me the missing number
How many fingers are there on two hands?
How many eggs are there in the box? How are they arranged? (in 2s)
Count the pairs of animals on the ark.
Count these pairs of socks. How many pairs are there? How many socks are there altogether?
How many buttons are there on this coat? Count them in twos. Now count them in fives. (answer - 10)
**EYFS 2 – Multiplication**  
**Progression (a combination of these models and images can be used for every objective)**

<table>
<thead>
<tr>
<th>To set out groups and find the total amount</th>
<th>To find matching groups that are the same amount</th>
<th>To recognise when each person is given the same amount</th>
<th>To double numbers to 5, then 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can recognise when objects are put into groups of the same amount.</td>
<td><img src="image1.png" alt="Image of matching groups" /></td>
<td><img src="image2.png" alt="Image of matching groups" /></td>
<td><img src="image3.png" alt="Image of double numbers" /></td>
</tr>
</tbody>
</table>

**To double quantities of objects**

- Using these toy chicks:
  - Can you double the chicks in the first box?
  - How many chicks now?
  - Can you double the cubes?

**To skip count in 2s**

- Count: Pairs of socks, pairs of shoes, pairs of gloves, pairs of animals, talk partners etc.

**To skip count in 5s**

- Objects that can be counted in pairs are even
- Objects that can’t be counted in pairs are odd

**To skip count in 10s**

- How many wheels on two cars?
- Double 4 = 8

**To place objects into arrays**

- How many objects?
- Double 4 = 8
## Year 1 – Multiplication
(When planning ensure you track back to Reception and forwards to year 2)

<table>
<thead>
<tr>
<th>National Curriculum</th>
<th>Notes and guidance (non-statutory)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.</td>
<td>Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities. They make connections between arrays, number patterns, and counting in twos, fives and tens.</td>
</tr>
</tbody>
</table>

### Key vocab:
- count in, double, halve, lots of, groups of, times, group in pairs, equal groups of, ×, times, multiplied by, multiple of, one, twice, three times, ten times, repeated addition, array, row, column.

### Key concepts
- Multiplication begins with counting patterns and equal groups in context.
- Multiplication is introduced when objects or numbers are combined. It is associated with the idea of repeated addition.
- Pupils will be able to conceptualise multiplication as groups of items.

### Potential barriers/misconceptions
- Still counts in ones to find how many there are in a collection of equal groups; does not understand vocabulary for example ‘multiplied by’.
- When objects placed in arrays it may be done inaccurately therefore link between arrays and answers unclear.
- Pupils may not focus on ‘rows of’ or ‘columns of’ but only see arrays as a collection of ones.
- Don’t understand how ‘turning the grid around’ shows that multiplication can be done in any order.

### Learning objectives (see over page for exemplification)
- To place objects into equal groups
- To double numbers
- To double two digit numbers
- To place objects into arrays
- To pictorially represent multiplication sentences
- To understand repeated addition
- Can describe an array in two ways
- To make multiplication stories
- To move towards the bar model to solve word problems

### Example Questions
- How many pencils do I need if everyone has to have 2? (There are eight people in the group)
- How many shoes do we need for these three dolls?
- John makes biscuits with 1 egg, 4 spoons of flour, 3 spoons of sugar, 5 spoons of milk. Lucy makes double the amount of biscuits. She will need: □ egg, □ spoons of flour, □ spoons of sugar, □ spoons of milk.
- Add more fives until the total is 25. 5 + 5 + ......
- Write the answer: 6 × 2 =

### Mental Maths
- To count in twos, fives and tens
- Count forwards and backwards in 2s from any given number.
- Count forwards and backwards in 5s from any given number.
- Count forwards and backwards in 10s from any given number.
- Recognition of all odd and even numbers
- Rapid recall of doubles to 10 (and corresponding halves)
- Rapid recall of doubles to 20
<table>
<thead>
<tr>
<th>Year 1 – Multiplication</th>
<th>Progression (a combination of these models and images can be used for every objective)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>To place objects into equal groups</strong></td>
<td><strong>To double numbers</strong></td>
</tr>
<tr>
<td>How many pencils in each group?</td>
<td>How many groups of shells are there?</td>
</tr>
<tr>
<td>Each group has 3 shells in.</td>
<td>How many shells are there in each group?</td>
</tr>
<tr>
<td>3 + 3 + 3 + 3 = 12</td>
<td>4 threes = 6</td>
</tr>
<tr>
<td>(4 groups)</td>
<td>(3 groups of 4 = 12)</td>
</tr>
<tr>
<td>How many altogether?</td>
<td>There are 12 shells altogether.</td>
</tr>
<tr>
<td>Can you count in 5s?</td>
<td><strong>To double numbers (over 10)</strong></td>
</tr>
<tr>
<td>5 + 5 + 5 = 15</td>
<td>Double 10 + 2</td>
</tr>
<tr>
<td><strong>To place objects into arrays</strong></td>
<td><strong>To pictorially represent multiplication sentences</strong></td>
</tr>
<tr>
<td>Arrange your objects into rows</td>
<td>5 x 2 = 10</td>
</tr>
<tr>
<td>Can you arrange them in different ways?</td>
<td>5 + 5 = 10</td>
</tr>
<tr>
<td>Each row must have the same number. Can you add the total amount?</td>
<td><strong>To understand repeated addition</strong></td>
</tr>
<tr>
<td>5 x 2 = 10</td>
<td><strong>To describe an array in two ways</strong></td>
</tr>
<tr>
<td><strong>To make multiplication stories</strong></td>
<td><strong>To move towards the bar model to solve word problems</strong></td>
</tr>
<tr>
<td>Can you tell a multiplication story about these cakes?</td>
<td>There are three children. Each child has five sweets. How many sweets do they have altogether?</td>
</tr>
<tr>
<td>I see □ groups of cakes. Each group has □ cakes. □ x □ = □ There are □ cakes altogether.</td>
<td>5 + 5 + 5 = 15</td>
</tr>
<tr>
<td>→ 5 x 3 = 15</td>
<td></td>
</tr>
<tr>
<td>3 x 5 = 15</td>
<td>They have 15 sweets altogether</td>
</tr>
</tbody>
</table>
# Year 2 – Multiplication (When planning ensure you track back to year 1 and forwards to year 3)

## National Curriculum
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

## Key vocabulary:
- count in, double, halve, lots of, groups of, times, group in pairs, equal groups of, x, times, multiplied by, multiple of, one, twice, three times, ten times, times as (big, long, wide... and so on) repeated addition, array, row, column.

## Key concepts:
- Double, times, multiply, multiplied by, multiple of, lots of, groups of, times as (big, long, wide...) X
- Multiplication is introduced when objects or numbers are combined.
- It is associated with the idea of repeated addition.
- Pupils will be able to conceptualise multiplication as groups of items.
- Multiplication can be done in any order- this can be shown in the arrangement of arrays.
- Doubling is the inverse of halving.

## Potential barriers/misconceptions
- Pupils may not focus on ‘rows of’ or ‘columns of’ but only see arrays as a collection of ones.
- Don’t understand how ‘turning the grid around’ shows that multiplication can be done in any order.
- Not understanding that multiplication is repeated addition.

## Learning objectives (see over page for exemplification)
- To identify odd and even numbers
- To understand multiplication as repeated addition
- To use arrays
- To know 2, 5, 10 times tables.
- To multiply using partitioning
- To understand the commutative property of multiplication.
- To interpret multiplication sentences (The first factor referring to the number of groups and the second factor as the number of items in each group.)
- To know all corresponding multiplication and division facts (i.e. 2×4=8, 4×2=8 and 8÷4=2, 8÷2=4)
- To break a number into factors
- To connect the 10 times table with place value
- To use the bar model to represent word problems

## Example Questions
- Respond rapidly to oral and written questions such as: two fives, double 5, 6 times 2, 5 multiplied by 2, multiply 4 by 2. Two tens, double 2, 3 times 4, 9 multiplied by two, multiply 5 by 8.
- Is 20 a multiple of 5? Yes
- 6×2=□ 9×□=18 □×□=14
- 6×10=□ 2x□=20 □×□=40
- How many wheels are there on three cars?
- Jo’s plane is 6cm wide. Mo’s box is twice as wide. How wide is Mo’s box? (scaling)
- Ella’s dad washes some cars. He uses 12 buckets of water. Each bucket has five litres of water. How many litres of water does he use altogether?
- Tara does not know how to work out 16 × 5. Can you show her how to do this?
- There are 15 apples in a tray. Ling has 4 trays of apples. How many apples does Ling have altogether?
- Show how you work it out.

## Key concepts
- Double, times, multiply, multiplied by, multiple of, lots of, groups of, times as (big, long, wide…) X
- Multiplication is introduced when objects or numbers are combined.
- It is associated with the idea of repeated addition.
- Pupils will be able to conceptualise multiplication as groups of items.
- Multiplication can be done in any order- this can be shown in the arrangement of arrays.
- Doubling is the inverse of halving.

## Mental Maths
- Rapid recall of 2, 5 and 10 times tables
- Count in 5s clockwise around a clock face/ anticlockwise around a clock face.
- Count forwards and backwards in 2s, 5s and 10s from any given number.
- Recognition of all odd and even numbers
- To recall related multiplication and division facts linked to other multiplication tables. (3×4=12, 4×3=12, 12÷4=3, 12÷3=4)
- Rapid recall of doubles and their corresponding halves. (double 12 is 24, half 24 is 12)
- Rapid recall of half of all 2 digit even numbers. (half of 12, 18, 42 etc)
- Recognise that multiples of 10 end in 0, 5 end in 5 and 0, 2 end in 0,2,4,6,8.
- Recognise two digit multiples of 10,5,2 (65 is a multiple of 5, 72 is a multiple of 2, 50 is a multiple of 5 and 10)
- Work out the four times table by doubling the two times table.
- Multiply a single digit by 2 or 10. ( 3×1 = 3, 7×10=70 etc)
- Multiply a single digit up to 5 by 2,3,4,5. (2×3=6 4×4=16 )
### Year 2 – Multiplication

**Progression** (a combination of these models and images can be used for every objective)

<table>
<thead>
<tr>
<th>To identify odd and even numbers</th>
<th>To understand multiplication as repeated addition</th>
<th>To use arrays</th>
</tr>
</thead>
<tbody>
<tr>
<td>All numbers that can be ‘paired’ are even. All numbers that are in the x2 table are even. All numbers that end in 0, 2, 4, 6, 8 are even. To establish if a number is even we look at the ones.</td>
<td>Three in a row Four in a column 3 + 3 + 3 = 9 3 x 3 = 9 3 groups of 3</td>
<td>Three in a row Four in a column 4 + 4 + 4 = 12 3 + 3 + 3 + 3 = 12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>To know 2, 5, 10 times tables up to x12.</th>
<th>To multiply using partitioning</th>
<th>To understand the commutative property of multiplication.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use concrete objects grouped, number lines, skip counting etc</td>
<td>Use knowledge of the 2, 5 and 10 times table to work out multiples of 7. 7 x 3 = (5 + 2) x 3 = (5 x 3) + (2 x 3) = 15 + 6 = 21</td>
<td>Using squared paper to support understanding Each child has picked 3 flowers. How many flowers will 5 children have altogether? The fixed number is 3 It is being multiplied by 5. (5 children will have 15 flowers altogether) If paper is turned it will show 5 x 3 = 15.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>To interpret multiplication sentences (The first factor referring to the number of groups and the second factor as the number of items in each group.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>How many books are there?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>To know all corresponding multiplication /division facts</th>
<th>To break a number into factors</th>
<th>To connect the 10 times table with place value</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 ÷ 2 x 4 = 8 4 ÷ 2 = 4 8 ÷ 2 = 4 8 ÷ 4 = 2 Part, part, whole.</td>
<td>12 12 12</td>
<td>0 ÷ 10 10 0 10 One ten 20 Two tens 30 Three tens 40 Four tens 50 Five Tens 60 Six Tens 70 Seven Tens 80 Eight Tens 90 Nine Tens 100 Ten Tens</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>To use the bar model to represent word problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>There are 5 biscuits in each packet. There are 4 packets. How many biscuits are there altogether?</td>
</tr>
</tbody>
</table>

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Chapter 4 - Multiplication
Year 3 – Multiplication (When planning ensure you track back to year 2 and forwards to year 4)

**National Curriculum**
- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

**Notes and guidance (non-statutory)**
- Pupils continue to practise their mental recall of multiplication tables when they are calculating mathematical statements in order to improve fluency.
- Through doubling, they connect the 2, 4 and 8 multiplication tables.
- Pupils develop efficient mental methods, for example, using commutativity and associativity (for example, \(4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240\)) and multiplication and division facts (for example, using \(3 \times 2 = 6, 6 \div 3 = 2\) and \(2 \times 3 = 6\)) to derive related facts (for example, \(30 \div 2 = 15, 60 \div 3 = 20\) and \(20 \div 3\)).
- Pupils develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division.
- Pupils solve simple problems in contexts, deciding which of the four operations to use and why. These include measuring and scaling contexts, for example, four times as high, eight times as long etc.) and correspondence problems in which m objects are connected to n objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).

**Key vocabulary**
- double, times, multiply, multiplied by, multiple of, lots of, groups of, times as (big, long, wide...) product, lots of, groups of, repeated addition, array, row, column.

**Key concepts**
- Multiplication as: repeated addition
- describing an array
- scaling (find a piece of string that is five times as long)
- Multiplication can be done in any order (\(5 \times 8 = 8 \times 5\)) but to understand that division cannot. \((16 \div 2 = \text{not the same as } 2 \times 16)\).
- Multiplication reverses division (is the inverse of division)
- When you multiply by ten the digits move one place to the left

**Potential barriers / misconceptions**
- Children may need to go back to multiplication as an array, or repeated addition to gain security.
- Some children struggle to apply partitioning and recombining when multiplying. E.g. 14 x3 is calculated as \((10 \times 3) + (4 \times 3) = 30 + 12 = 42\)
- Lack of confidence with place value sees confusion in the value of the two digits.
- Children are incorrectly taught that x10 involves ‘adding a zero’ rather than developing understanding of place value. Also unable to see that x100 is the same as x10 and x10 again.

**Example questions**
- Respond rapidly to oral and written questions such as: three fives, double 11, 6 times 4, 5 multiplied by 8, multiply 4 by 3. Four tens, double 24, 3 times 5, 9 multiplied by four, multiply 5 by 8.
- Is 20 a multiple of 5? 5x3 □ 8□x=40 □ x=9=45 6x20 □ \(\Delta x=60\)
- In the shop there are 6 cans of beans in each of 4 rows. How many cans of beans are there?
- Natalie has 10 football cards. Kojo has 3 football cards for every one of Natalie’s. How many cards does Kojo have?
- Finn has 4 stickers. Howard has 3 times as many stickers as Finn. How many stickers does Howard have?

**Learning objectives**
- To use number bonds for factors and products
- To understand how place value changes when multiplying by 10
- To calculate two-digit numbers multiplied by one-digit numbers
- To carry out short multiplication without regrouping
- To understand measuring and scaling problems

**Mental Maths**
- Rapid recall of 3, 4 and 8 times tables
- Count forwards and backwards in 3s from any given number.
- Count forwards and backwards in 4s from any given number.
- Count forwards and backwards in 8s from any given number.
- To use the 2, 5 and 10 times table to derive other multiplication facts (if I know 2x5=10 I also know 20x5=100)
- To know doubles of all numbers up to 50
- To know doubles of all multiples of 5 up to 100
- To know doubles of all multiples of 5 up to 100
- Observe the effect of multiplying by 10
- Multiply any single digit by 1, 10, 100 and 0
- Multiply a two-digit number by 2, 3, 4, or 5 without crossing the tens boundary. \((11x5, 23x2)\)
- Check halving with doubling
- To multiply multiples of 10 with 1 digit number

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### Year 3 – Multiplication Progression (a combination of these models and images can be used for every objective)

<table>
<thead>
<tr>
<th>To use number bonds for factors and products</th>
<th>To understand how place value changes when multiplying by 10</th>
<th>To calculate two digit numbers multiplied by one digit numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>(factor ÷ factor = product) 6 x 3 = □</td>
<td>64 x 10 = 640</td>
<td>24 x 3= <img src="image" alt="Visual link to arrays" /></td>
</tr>
<tr>
<td>And 4 x 5 = 20</td>
<td>When multiplying by ten the original number moves one place to the left. It gets ten times bigger (X10). Zero acts as a place holder in the ones.</td>
<td><img src="image" alt="Visual link to arrays" /></td>
</tr>
<tr>
<td><img src="image" alt="number bonds" /></td>
<td><img src="image" alt="array" /></td>
<td><img src="image" alt="visual link to arrays" /></td>
</tr>
</tbody>
</table>

**To carry out short multiplication without regrouping**

<table>
<thead>
<tr>
<th>12 x 3</th>
<th>First multiply the ones by 3</th>
<th>Then multiply the tens by 3</th>
<th>68 x 2=</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="tens ones" /></td>
<td><img src="image" alt="tens ones" /></td>
<td><img src="image" alt="tens ones" /></td>
<td><img src="image" alt="tens ones" /></td>
</tr>
<tr>
<td>1 2 6</td>
<td>X 3</td>
<td>X 3</td>
<td>68 1 3</td>
</tr>
<tr>
<td>2 ones x 3 = 6 ones</td>
<td></td>
<td>1 ten x 3 = 3 tens</td>
<td></td>
</tr>
</tbody>
</table>

**When we multiply 12 by 3, we get the product of 12 and 3. (36 is the product of 12 and three)**

**To understand measuring and scaling problems**

Beccy read 750 words in one day. Izabela read three times as many words. How many pages did Izabela read?

- 750 x 3 = 2250  
  izabela read 2250 words

Mr Miaow had 8 bags of cat treats. Each bag contained 156 cat treats. He fed 382 cat treats to his cats. How many cat treats had he left?

- 156 x 8 = 1248
- 1248 – 382 = 866 Mr Miaow had 866 cat treats left.

<table>
<thead>
<tr>
<th><img src="image" alt="tens ones" /></th>
<th><img src="image" alt="tens ones" /></th>
<th><img src="image" alt="tens ones" /></th>
<th><img src="image" alt="tens ones" /></th>
</tr>
</thead>
<tbody>
<tr>
<td>156 1248 treats</td>
<td>?</td>
<td>382 treats</td>
<td>?</td>
</tr>
</tbody>
</table>

First, multiply the ones by 2  
6 8  
8 ones x 2 = 16 ones  
Regroup the ones:  
16 ones = 1 ten and 6 ones

Then, multiply the tens by 2  
6 8  
6 tens x 2 = 12 tens  
Add the tens:  
12 tens + 1 ten = 13 tens  
Regroup the tens:  
13 tens = 1 hundred 3 tens

So…. 68 x 2 = 136
**Year 4 – Multiplication** (When planning ensure you track back to year 3 and forwards to year 5)

### National Curriculum
Recall multiplication and division facts for multiplication tables up to 12 × 12
Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
Recognise and use factor pairs and commutativity in mental calculations
Multiply two-digit and three-digit numbers by one-digit numbers using a formal written layout
Solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

### Notes and guidance (non-statutory)
Pupils continue to practise recalling and using multiplication tables and related division facts to aid fluency.
Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6).
Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers.
Pupils write statements about the equality of expressions (for example, use the distributive law 39 × 7 = 30 × 7 + 9 × 7 and associative law (2 × 3) × 4 = 2 × (3 × 4)). They combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations for example, 2 x 5 x 5 = 10 x 6 = 60.
Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers.

### Key concepts
When you multiply by ten the digits move one place to the right
That commutative law is: 6 × 15 = 15 × 6
Associative law is: 6 × 15 = (6 × 5) × 3 = 30 × 3 = 90
Distributive law is: 18 × 5 = (10 + 8) × 5 = (10 × 5) + (8 × 5) = 50 + 40 = 90
56 + 56 + 56 is equivalent to 56 × 3 or 3 × 56
Multiplication by 1 leaves a number unchanged
Multiplication by zero results in zero
Multiplication is the inverse of division

### Key vocab:
- double, times, multiply, multiplied by, multiple of, lots of, groups of, times as (big, long, wide...), product, lots of, groups of, repeated addition, array, row, column, factor, inverse.

### Potential barriers/ misconceptions
Children unaware of language of multiplication: multiple, factor
Children not secure in their rapid recall of times tables facts and conceptual understanding
Some children not using known facts and commutative properties to solve sums such as 8 × 2 = 16
Children may not see the link between known facts and multiplying multiples by 10 and 100.
Children are incorrectly taught that × 10 involves ‘adding a zero’ rather than developing understanding of place value. Also unable to see that 1 × 100 is the same as 10 x 10 and 10 x 10 again.

### Example Questions
How many times larger is 250 than 25? Prove it.
How many £1 coins are there in £15, £150, £1500? How many 10p coins?
Tins of cat food are put in packs of 10. One tin costs 62p. How much does one pack cost? Ten packs?
Respond rapidly to oral questions explaining strategy: two elevens, double 16, seven times six, 9 multiplied by 3, multiply 15 by 7, by zero, by 1.
Is 40 a multiple of 5? How do you know? What else is it a multiple of? What is the product of 12 and 7?

### Learning objectives (see over page for exemplification)
- To multiply by ten using place value grids and dienes
- To multiply two digit numbers by a one digit number (see year 3 exemplification)
- To multiply three digit numbers by one digit number
- To multiply two digit by two digit number
- To use the distributive law: 32 x 3 = (30 x 3) + (2 x 3) = 90 + 6 = 96
- To use associative law to multiply three numbers
- To solve problems using scaling
- To derive multiplication and division facts from three digit numbers
- To solve two step word problems.
- To recognise factors of a number
- To multiply decimals

### Mental Maths
- Rapid recall of all numbers multiplied by 10, 100, 1000
- Rapid recall of all multiplication and division facts up to 12 × 12
- To understand what happens when multiplying by 1 and 0
- To multiply together three numbers
- To know by heart all doubles and halves (double 34 is double 30 + double 4 = 60 + 8 = 68)
- To multiply by 4, double and double again: 7 × 4 = double 7 = 14. Double 14 = 28
- To multiply by 5 (multiply by 10 and halve: 5 × 9 = 10 × 9 = 90 halved = 45)
- To multiply by 20 (multiply by 10 and double)
- Work out 8 times table by doubling four times table.
- Use doubling to work out multiples of 15. (1 × 15 = 15, 2 × 15 = 30, 4 × 15 = 60, 8 × 15 = 120, 16 × 15 = 240) Use combinations of these facts to find e.g. 11 × 15 = (8 × 15 + 2 × 15) + 1 × 15 = 120 + 30 + 15 = 165
- Work out the six times table by adding 2 times table facts and 4 times table facts.
- To multiply a number by 9 or 11, multiply it by 10 and add/subtract the number (14 × 9 = 140 - 14 = 126 and 14 × 11 = 140 + 14 = 154)
- To know the three corresponding number facts when given a multiplication number sentence.

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Primary Advantage Maths Programme 2015
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Chapter 4 - Multiplication
Year 4 – Multiplication

Progression (a combination of these models and images can be used for every objective)

To multiply three digit numbers by one digit number

656 x 2 =

6 ones x 2 = 12 ones
12 ones = 1 Ten and 2 ones

5 tens x 2 = 10 tens
Add the tens: 10 tens + 1 ten = 11 tens
6 hundreds x 2 = 12 hundreds
Add the hundreds: 12 hundreds + 1 hundred = 13 hundreds

Regroup the hundreds:
13 hundreds = 1 thousand and 3 hundreds

Regroup the tens:
11 tens = 1 hundred 1 ten
Add the tens: 10 tens + 1 ten = 11 tens
Add the hundreds: 12 hundreds + 1 hundred = 13 hundreds

To multiply by ten and one hundred

64 x 10 = 640
64 x 100 = 6400
When multiplying by one hundred the original number moves one place to the left.
It gets one hundred times bigger (x100).
Zero acts as a place holder in the ones.

64 x 100 = 6400

56 x 27 = 1512

18 x 13 = 234
18 x 3 = 54
18 x 10 = 180
56 x 27 = 1512

To use the distributive law: 32 x 3 = (30 x 3) + (2 x 3) = 90 + 6 = 96
To use associative law to multiply three numbers

(2 x 3) x 4 = 24
2 x (3 x 4) = 2 x 12 = 24
(3 x 4) = 12
6 x 4 = 24
2 x 12 = 24

John earns £125 a week – his brother earns three times as much = £125 x 3
I purchased 25 tickets at £12 per ticket = 25 x 12
A box of drinks has 24 cans. Each can costs 45p each = 24 x 45

To solve two step word problems. (including inter scaling)

At the Olympic stadium the number of men is 3 times the number of women. The number of women is 5 times the number of children at the stadium.
1. How many times the number of children is the number of men?
3 x 5 = 15 The number of men is 15 times the no. of children.
2. If there are 730 children how many men are there?
730 x 15 = 10 950. There are 10 950 men

‘24 is a multiple of 6 then 6 is a factor of 24’.
We colloquially say ‘6 goes into’ 24 four times’.

Factors of 24 are:
1, 2, 3, 4, 6, 12 and 24

Arrays support understanding of factor pairs: 1x24, 2x12, 3x8, 4x6

To recognise factors of a number

0.2 x 3 = 0.6
0.2 + 0.2 + 0.2 = 0.6
3 x 2 tenths = 6 tenths = 0.6

To multiply decimals

0.2
X 3
0.6
Year 5 - Multiplication

National Curriculum
Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers know and use the vocabulary of prime numbers, prime factors and composite (nonprime) numbers
Establish whether a number up to 100 is prime and recall prime numbers up to 19
Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
Multiply whole numbers and those involving decimals by 10, 100 and 1000
Recognise and use square numbers and cube numbers, and the notation for squared (\(^2\)) and cubed (\(^3\))
Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Key vocabulary: times, multiply, multiplied by, product, multiple, inverse and \(x\) sign, double, times, multiply, multiplied by, multiple of, lots of, groups of, times as (big, long, wide...) product, lots of, groups of, array, row, column, factor.

Key concepts: When you multiply a number by 10/100, the digits move one/two places to the left
Multiplying by 100 is equivalent to multiplying by 10 and 10 again
That commutative law is: \(8 \times 75 = 75 \times 8\)
Associative law is: \(18 \times 11 = (2 \times 9) \times 11 = 2 \times (9 \times 11) = 2 \times 99 = 198\)
Distributive law is: \(26 \times 7 = (20+6) \times 7 = (20 \times 7) + (6 \times 7) = 140 + 42 = 182\)
With positive whole numbers, multiplication makes a number larger
Multiplication is the inverse of division and can be used to check results

Children may struggle to partition a two digit number into tens and units correctly for whole numbers and tenths for decimals.
Children believe that multiplication always increases a number. This is only when a positive number is multiplied by a whole number greater than 1.

Example questions:
How many times larger is 2600 than 26? Prove it.
How many £10 notes are there in £120, £1200? How many £1 coins, 10p coins, 1p coins?
Tins of cat food are put in packs of 10. One tin costs 62p. How much does one pack cost? Ten packs?
What is 4 squared? What is the square of 6? What is 8? Which number multiplied by itself is 36?
Respond rapidly to: two twelves, double 32, 7 times 8, 9 multiplied by 7, multiply 31 by 8, zero, 1.
Is 81 a multiple of 3? How do you know? What is the product of 25 and 4?
132x6
6x=4.8
80x9=189
90x9=810
6x=9=18

I have 16 boxes each weighing 20.5kg. What is their total weight?
What is 72 multiplied by 38?
Calculate 349 x 6
What is the area of a field 27 metres by 37 metres?

Learning objectives (see over page for exemplification)
To identify common factors of two numbers.
To know prime numbers, prime factors and composite (non-prime) numbers
To solve problems involving multiplication.
To multiply numbers up to four digits by a one digit number
To multiply numbers up to four digits by a two digit number
To recognise and use squared and cubed numbers
To understand the law of distributivity
To multiply whole numbers & decimals by 10, 100, 1000

Mental Maths
To find all factor pairs of a number & find common factors of two numbers
To establish whether a number up to 100 is prime
To recall prime numbers up to 19
Recognise 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 as square numbers (relate to drawings of squares)
Find all the pairs of factors for any number to 100 (pairs of factors to 36 are 18x3, 12x2, 36x1, 18x2, 36x1, 18x1, 36x1, 18x1, 36x1)
Use factors for finding products mentally (16 \(\times\) 12 = 16 \(\times\) 3 \(\times\) 2 \(\times\) 2 = 48 \(\times\) 2 \(\times\) 2 = 96 \(\times\) 2 = 192)
To double using known facts (double 79 = double 70 + double 9 = 140 + 18 = 158)
Double a number ending in 5 and halve the other number (16 \(\times\) 5 is equivalent to 8 \(\times\) 10 = 80)
To multiply by 50 (multiply by 100, then halve: 26 \(\times\) 50 = 26 \(\times\) 100 = 2600 halved = 1300)
Calculate 25 times table by doubling: (1x25) \(\times\) 2 \(\times\) 2 \(\times\) 25 \(\times\) 100, 8 \(\times\) 25 \(\times\) 100, 16 \(\times\) 25 \(\times\) 100 use combinations of these facts to work out e.g. 25 \(\times\) 25 = (16 \(\times\) 25) + (8 \(\times\) 25) + (1 \(\times\) 25) = 625
Calculate combinations of these facts to work out e.g. 25 \(\times\) 25 = (16 \(\times\) 25) + (8 \(\times\) 25) + (1 \(\times\) 25) = 625
Work out 12 times table by adding 2 times table and 10 times table
To multiply a number by 19 or 21, multiply it by 20 and add or subtract the number (13 \(\times\) 21 = 13 \(\times\) 20 + 3 = 273)
### Year 5 – Multiplication

**Progression (a combination of these models and images can be used for every objective)**

<table>
<thead>
<tr>
<th>To identify common factors of two numbers.</th>
<th>To know prime numbers, prime factors and composite (non-prime) numbers.</th>
<th>To solve problems involving multiplication.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Any number that has <strong>two</strong> factors and no more is a prime number. (It cannot be divided by any number except one and itself) The first 20 prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67 and 71. (one is not prime as it has only 1 factor)</td>
<td>Every number can be written as a product of prime numbers: 40 = 2 x 2 x 2 x 5 126 = 2 x 3 x 3 x 7 28 = 2 x 2 x 7 e.g. 10 has more than 2 factors: 1, 2, 5, and 10. It is called a composite number and can be arranged as a rectangular array with more than one row. (Prime numbers can only be arranged in a single row) Composite numbers are often called rectangular numbers. (links to arrays) All non-prime numbers (except 1) are composite.</td>
<td></td>
</tr>
<tr>
<td><strong>What are the common factors of 8 and 12?</strong></td>
<td>40 = 2 x 2 x 2 x 5 126 = 2 x 3 x 3 x 7 28 = 2 x 2 x 7 e.g. 10 has more than 2 factors: 1, 2, 5, and 10. It is called a composite number and can be arranged as a rectangular array with more than one row. (Prime numbers can only be arranged in a single row) Composite numbers are often called rectangular numbers. (links to arrays) All non-prime numbers (except 1) are composite.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The factors of 8 are:</th>
<th>The factors of 12 are:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2, 4, and 8</td>
<td>1, 2, 3, 4, 6, and 12</td>
</tr>
<tr>
<td>The common factors of 8 &amp; 12 are: 1, 2 and 4.</td>
<td></td>
</tr>
</tbody>
</table>

**To understand the law of distributivity**

Distributivity can be expressed as: $a(b + c) = ab + ac$

<table>
<thead>
<tr>
<th>Distributivity can be expressed as:</th>
<th>To multiply whole numbers &amp; decimals by 10, 100, 1000</th>
<th>Multiplying decimals by 100 and 1000:</th>
</tr>
</thead>
<tbody>
<tr>
<td>If: a=12, b=5, c=2 12 (5 + 2) = 12 x 7 = 84 = (12 x 5) + (12 x 2) = 60 + 24 = 84</td>
<td>5.928 x 10 = 59.28 X10</td>
<td>0.3 x 100 = $\frac{3}{10}$ x 100 0.3 x 1000 = $\frac{3}{100}$ x 1000</td>
</tr>
</tbody>
</table>

### Example Problems

**Tom had £37 56. He saved £650 and spent the rest on 12 pairs of shoes and some clothes. The pairs of shoes cost £205 each. How much did he spend on clothes?**

- £37 56
- £37 56 - £650 = £3106
- She spent £3106 altogether
- 12 x £205 = £?
- £3106
- 12 pairs of shoes cost £
- £ 3106
- £3106 - £650 = £2456
- He spent £2456 on clothes.

**To recognise and use squared numbers**

$3^2$ means ‘3 squared’, or 3 x 3. It tells us how many times we should multiply 3 by itself.

<table>
<thead>
<tr>
<th>$2^2$</th>
<th>$3^2$</th>
<th>$4^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 1</td>
<td>2 x 2</td>
<td>3 x 3</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

1, 4, 9, 16, 25... are known as square numbers.

**To recognise and use cubed numbers**

$2^3$ means ‘2 cubed’, and is written as $2^3$.

<table>
<thead>
<tr>
<th>$1^3$</th>
<th>$2^3$</th>
<th>$3^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 1 x 1</td>
<td>2 x 2 x 2</td>
<td>3 x 3 x 3</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>27</td>
</tr>
</tbody>
</table>

1, 8, 27, 64, 125... are known as cube numbers.

**To understand factor, multiple, prime, square and cube numbers and construct equivalence statements.**

$4 x 35 = 2 x 2 x 35$

$3 x 270 = 3 x 3 x 9 x 10 = 9^2 x 10$

### Example Problems

**To multiply numbers up to four digits by a one digit number (short multiplication)**

- $2741 x 6 = 16446$
- $286 x 29 = 8294$

**To multiply numbers up to four digits by a two digit number (long multiplication)**

- The same method would be applied for four digit multiplication.

**To understand the law of distributivity**

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<tr>
<th>Distributivity can be expressed as:</th>
<th>To multiply whole numbers &amp; decimals by 10, 100, 1000</th>
<th>Multiplying decimals by 100 and 1000:</th>
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<td>5.928 x 10 = 59.28 X10</td>
<td>0.3 x 100 = $\frac{3}{10}$ x 100 0.3 x 1000 = $\frac{3}{100}$ x 1000</td>
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**Multiplying decimals by 100 and 1000:**

<table>
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<th>Multiplying decimals by 100 and 1000:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.3 x 100 = \frac{3}{10} x 100$</td>
</tr>
<tr>
<td>$0.3 x 1000 = \frac{3}{100} x 1000$</td>
</tr>
<tr>
<td>$= 3 x 10 = 30$</td>
</tr>
<tr>
<td>$= 3 x 100 = 300$</td>
</tr>
</tbody>
</table>
**Year 6 – Multiplication** (When planning ensure you track back to Year 5 for progression)

**National Curriculum**
- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.
- Perform mental calculations, including with mixed operations and large numbers
- Identify common factors, common multiples and prime numbers
- Use their knowledge of the order of operations to carry out calculations involving the four operations
- Solve problems involving addition, subtraction, multiplication and division
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

**Key Vocab:** times, multiply, multiplied by, product, multiple, inverse and x sign, double, multiple of, lots of, groups of, times as (big, long, wide…) product, lots of, groups of, repeated addition, array, row, column.

**Key Concepts**
- When you multiply by 10/100/1000 the digits move 1/2/3 places to the left and get 10x/100x/1000x bigger
- That commutative law is: 95x78 = 78x95
- Associative law is: 10.4x40 = 10.4 x (10x4) or (10.4x10) x 4
- Distributive law is: 46x98 = 46 x (100-2) = (46x100) - (46x2) = 4600 - 92 = 4508
- Multiplication is the inverse of division

**Potential Barriers/Misconceptions**
- Misconceptions can arise when multiplying decimals. Belief that if 1x1=1 then 0.1x0.1=0.1 (this is ⅟10 x⅟10 which is one tenth ‘of’ one tenth which =1/100= 0.01)
- Interpreting a multiplication number sentence
  - 2x6 is often interpreted as the same as 6x2.
  - 2x6 is ‘2 multiplied by 6’, 2 taken six times or 2+2+2+2+2+2. The first number is ‘operated on’.
  - 6x2 would be ‘6 multiplied by 2’, 6 taken twice or 6+6.
  - ‘Everyday’ interpretation (referred to throughout programme) can be different as 2x6 is referred to as 2 ‘times’ 6 or 2 ‘groups of’ 6. This is not a problem as it is commutative and both will give the answer 12.

**Example Questions**
- How many times larger is 26000 than 2?
- How do you know? What is the product of 125 and 4?
- How many £100 notes are there in £1200, £12 000, £120, 000? How many £10 notes?
- How many £1 coins, 10p coins, 1p coins?
- 132x46=□
- 38x□=190
- □ x9=18.9
- How many tins of cat food cost 62p. They are put in packs of 10.
- Ten packs are put in a box.
- Ten boxes are put in a crate.
- How much does one crate cost?
- 10 crates? 100 crates?
- How many £100 notes are there in £1200, £12 000, £120, 000? How many £10 notes?
- How many £1 coins, 10p coins, 1p coins?
- How many £100 notes are there in £1200, £12 000, £120, 000? How many £10 notes?
- How many £1 coins, 10p coins, 1p coins?
- How many £100 notes are there in £1200, £12 000, £120, 000? How many £10 notes?
- How many £1 coins, 10p coins, 1p coins?
- How many £100 notes are there in £1200, £12 000, £120, 000? How many £10 notes?
- How many £1 coins, 10p coins, 1p coins?
- How much does one crate cost?
- 10 crates? 100 crates?
- How many £100 notes are there in £1200, £12 000, £120, 000? How many £10 notes?
- How many £1 coins, 10p coins, 1p coins?
- How many £100 notes are there in £1200, £12 000, £120, 000? How many £10 notes?
- How many £1 coins, 10p coins, 1p coins?
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- How many £100 notes are there in £1200, £12 000, £120, 000? How many £10 notes?
- How many £1 coins, 10p coins, 1p coins?
**Chapter 4 - Multiplication**

**Progression** (a combination of these models and images can be used for every objective)

<table>
<thead>
<tr>
<th>To multiply multi digit numbers up to 4 digits by a two digit whole number</th>
<th>To multiply fractions</th>
<th>To multiply decimals</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{8} \times \frac{8}{9} = \frac{3 \times 8}{4 \times 9}$</td>
<td>$\frac{3}{5} \times \frac{1}{2} = \frac{3 \times 1}{5 \times 2}$</td>
<td>$1 \times 10 = 10$</td>
</tr>
<tr>
<td>product $= \frac{2}{3}$</td>
<td>$\frac{3}{10}$</td>
<td>$0.1 \times 10 = 1$</td>
</tr>
<tr>
<td>$\frac{2}{3}$</td>
<td></td>
<td>$11 \times 10 = 110$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.11 \times 10 = 1.1$ (see below)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$111 \times 10 = 1110$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.111 \times 10 = 1.11$</td>
</tr>
</tbody>
</table>

To understand the commutative, distributive and associative laws of multiplication:

- Commutative laws of multiplication: $a \times b = b \times a$
- Associative law of multiplication: $(a \times b) \times c = a \times (b \times c)$
- Distributive laws of multiplication: $(a + b) \times c = (a \times c) + (b \times c)$
  \[(a - b) \times c = (a \times c) - (b \times c)\]

- $5 \times 28 = 28 \times 5$ (commutative law)
- $28 \times 5 = 28 \times 2 + 28 \times 2$ (associative law)
- $(14 \times 2) \times 5 = 14 \times (2 \times 5)$ (associative law)
- Or $(20 + 8) \times 5 = (20 \times 5) + (8 \times 5)$

- Or we could choose to think of the $28$ as $30 - 2$ then distribute the multiplication by $5$ across this subtraction:
  \[30 \times 5 = 150 \text{ and } 2 \times 5 = 10 \text{ therefore } 150-10=140\] (distributive)

To relate common factors to equivalent fractions

<table>
<thead>
<tr>
<th>Brackets</th>
<th>Indices</th>
<th>Division</th>
<th>Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Additions</td>
<td>Subtractions</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To solve word problems involving all four operations

A pile of books is $4.55$cm high. Find the height of 9 books.

Round off the answer to 1 decimal place.

- $0.65 \times 9 = 5.85$ (rounded off is 5.9)

The height of nine books is 5.9 cm.

- $4.55 + 7 = 0.65$